

$$\frac{v}{v_0} = 1 + \left(\frac{\rho_0}{\rho} - 1 \right) \frac{1 + \mu}{3(1 - \mu)},$$

which governs the relation between the relative volume v/v_0 of the cylinder in a rigid yoke, on the one hand, and the relative density ρ/ρ_0 ($\rho_0 = 1/v_0$) of the free body from the same material, on the other, for a given temperature t when there are no external forces ($p = 0, \sigma = 0$).

It follows from Fig. 2 that the computed and experimental results are in good agreement. These data can be used to determine the dependence of the elastic moduli, the coefficient of thermal expansion, and some other physical characteristics as a function of the pressure and temperature.

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ANALYSIS OF THERMAL MODEL OF THE CONTACT HEAT TRANSFER OF ROUGH SURFACES

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A thermal model of the contact heat transfer between rough surfaces is considered, taking into account curvature of the current lines in the gaps. Theoretical relations determining the contact thermal resistance at small pressures are obtained.

Formulation of the Problem

One of the parameters which has a significant effect on the thermal conditions in apparatus is the contact thermal resistance (CTR) due to imperfections of the mechanical connection between the contacting surfaces.

In [1-4] a detailed analysis was made of the results of investigations of CTR by Soviet and non-Soviet authors, the mechanism of contact was explained, the physical basis of the heat transfer through the contact zone was discovered, and practical recommendations for the intensification of heat transfer were given. However, as the forms of real mechanical connections are so different and so complex, it is often a laborious task to use the results of [1-4] for the calculation of CTR. There are several reasons for this:

a) the theoretical relations are only adequately reliable for the simplest case of contacting-object geometry - tangency of plane surfaces;

b) the range of specific compression forces of the contacting surfaces investigated is not characteristic for typical problems of instrument-making. The theoretical dependences allow CTR to be determined at pressures no less than $(10-60) \cdot 10^5 \text{ N/m}^2$. In practice, instrument-making usually requires calculations of CTR at lower pressures – $(1-10) \cdot 10^5 \text{ N/m}^2$. The recommendations for heat-transfer intensification in the contact zone available in the literature give only qualitative characteristics. The results given below allow the existing gaps to be somewhat reduced.

The heat flux P passing through the contact zone may be divided, for the purpose of analysis, into two components: a part P_p of the flux passes through the point of physical contact, and a part P_m through the medium. The corresponding conductivities are denoted by α_p and α_m , and the total conductivity is expressed as their sum

$$\alpha = \alpha_p + \alpha_m \quad (1)$$

The first term α_p in Eq. (1) is determined by the actual contact area and the thermal conductivity of the contacting materials. There are several methods of calculating α_p . The most widespread method which is sufficiently accurate is derived from the realization of the "button" model [5]. Here the contact is modeled using two infinite cylinders with a single circular contact spot at the center, the area of which models the actual contact spot.

The actual contact area (ACA) η depends on the physical properties of the materials, their treatment, and the compression forces. The error in calculating α_p is determined to a considerable extent by the error in η , and therefore the method for calculating α_p may be improved both by making the model of heat transfer through the contact zone more accurate and by improving the method of calculating η . In [5, 6], the present problem is discussed in sufficient detail and, although the existing solutions cannot be regarded as final, they may nevertheless be used for the calculation.

The value of the second component in Eq. (1) is determined by the configuration of the contact-medium surfaces. In [1-3], the following relation is proposed for the calculation of α_m :

$$\alpha_m = \lambda_m / \delta_{\text{equ}} \quad (2)$$

where δ_{equ} is the thickness of the equivalent plane layer applied between the contacting media, and is determined by equating the volumes of the effective layer and the real gap

$$\delta_{\text{equ}} = \frac{S_N}{\int \delta(S_N) dS_N + 2l_T} \quad (3)$$

where l_T is the value of the temperature jump if the layer is filled with gas; S_N , nominal area; $\delta(S_N)$, size of the gap, which varies with S_N .

Analysis of Methods of Calculating CTR

In the literature there has been fairly detailed consideration of the heat transfer at specific pressures $P > (10-50) \cdot 10^5 \text{ N/m}^2$, i.e., in the case when the heat transfer is determined by the conductivity through the point of physical contact, $\alpha_p > \alpha_m$. This situation is explained by the historical development of these investigations. The first and most fundamental works arose out of engineering fields associated with turboconstruction, atomic power, and rocket and aeronautical engineering. The mechanical and thermal models on which the calculation of α_p is based have been sufficiently well studied [1, 2]. The conductivity α_m plays the role of a correction here, and has been less thoroughly studied than α_p . In instrument-making, the main contribution to the heat transfer through the contact area is often the conductivity of the intercontact medium $\alpha_m \geq \alpha_p$, which is explained both by the small specific pressure $(1-50) \cdot 10^5 \text{ N/m}^2$ and by the use of greases and pastes with high thermal conductivity $\lambda = (0.1-3) \text{ W/m} \cdot \text{K}$.

It was also noted that on changing the medium filling the intercontact region the conductivity α_m does not change in proportion to the change in thermal conductivity of the medium

$$\frac{\alpha_{1m}}{\alpha_{2m}} = \frac{\alpha_{c1} - \alpha_{p1}}{\alpha_{c2} - \alpha_{p2}} \neq \frac{\lambda_{1m}}{\lambda_{2m}}$$

Here α_{c1} , α_{c2} are the total contact conductivities for different filling media with thermal conductivities λ_{1m} , λ_{2m} ; α_{p1} , α_{p2} are the conductivities through the contact point; α_{1m} , α_{2m} are the conductivities through the medium.

In the light of the above factors, the model of heat transfer was refined, and the appropriate changes

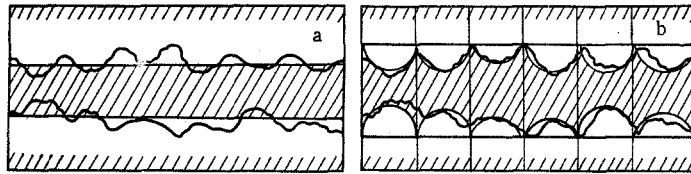


Fig. 1. Real profiles: a) replaced by an equivalent plane layer; b) by a set of elementary volumes.

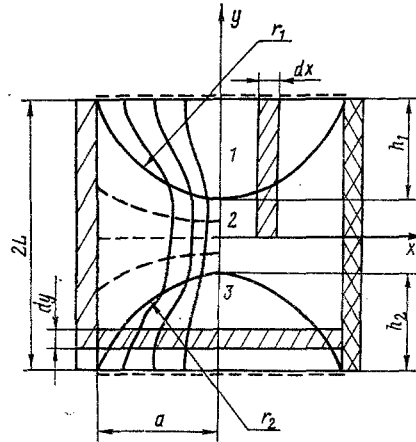


Fig. 2. Elementary volume and method of dividing auxiliary surfaces.

were made in the theoretical relations determining the CTR; this affected primarily the determination of α_m .

In the known methods [1, 2] for determining α_m , it is assumed that the contacting surfaces are isothermal. Below, a thermal model which takes into account that the surfaces are nonisothermal in determining α_m is proposed.

Geometric Model of Contact

The characteristics of the sample-surface profile (height and radius of curvature at the top of the microprojections) depend on the treatment and properties of the material. A typical surface profile is shown in Fig. 1a. In the general case, such microprojections take the form of segments of an ellipsoid, of which the height is most often less than the radius of curvature at the top. In [5], a comparative analysis of different models was made, and it was shown that the microprojections are most expediently modeled as a set of spherical segments distributed at constant density over the surface (Fig. 1b) of the contacting surfaces. The height of an individual microprojection is a random quantity conforming to a normal distribution law. The axes of opposing microprojections of contacting surfaces lie on a single line.

Analogous models are often used in investigating the actual contact area, closest approach, frictional coefficient, and thermal conductivities of mixtures and composite materials,

Thus, the contacting surfaces are represented by a set of elementary volumes, shown in Fig. 2. Here r_1 and r_2 are the radii of curvature; $2L$ is the maximum height of the volume; h_1 and h_2 are the heights of the segment.

The height and base diameter of the segment correspond to the spacing and height of the microprojections. The contacting surfaces usually have different roughnesses, and therefore the base diameters a_1 and a_2 are not equal. Different methods of choosing the mean a may be considered. Analysis results in the following recommendation

$$a = \frac{2a_1a_2}{a_1 + a_2} \quad (4)$$

Thus, each elementary volume forms a cylinder whose side surface does not intersect the current lines and whose base is an isothermal plane. The contacting surfaces form a set of elementary volumes. The seg-

ments in the cells are of different heights. Depending on the form of treatment, the distribution law may be different. At present, many researchers believe [6, 7] that the height distribution of the microprojection tops follows a normal law

$$f(h_i) = \frac{1}{\sigma_i \sqrt{2\pi}} \exp \left[-\frac{(h_i - m_i)^2}{2\sigma_i^2} \right], \quad (5)$$

where h_i is the height of the microprojection; m_i , mathematical expectation of a random quantity; σ , mean square deviation; $i = 1, 2$ denotes the first and second surface, respectively.

Investigations [4-7] of the contact interaction of solids have shown that the height distribution of the microprojections may be described by a normal law. In view of this, it is assumed on the basis of the "three-sigma" approach [8] that

$$m = h_{\max i}/2; \sigma = h_{\max i}/6. \quad (6)$$

Further, it may be confidently asserted that the difference $h_{\max i} - h_i$ between the constant quantity $h_{\max i}$ and the random quantity h_i is also random, obeying a normal distribution law with the same parameters m and σ . Then $\delta = (h_{\max 1} - h_1) + (h_{\max 2} - h_2)$ is the gap between roughness microprojections. The parameter δ in the model is also a random quantity obeying the normal distribution law. An expression for the probability density of the given random quantity may be obtained by considering the combination of the two normal distribution laws for each side

$$g(\delta) = \frac{1}{\sigma_\delta \sqrt{2\pi}} \exp \left[-\frac{(\delta - m_\delta)^2}{2\sigma_\delta^2} \right], \quad (7)$$

where δ is the size of the gap; σ_δ and m_δ are the mean square deviation and mathematical expectation, defined as follows:

$$\sigma_\delta = \sqrt{\sigma_1^2 + \sigma_2^2}; m_\delta = m_1 + m_2.$$

For contact between surfaces of the same roughness, the gap-thickness probability density is given by the expression

$$g(\delta) = \frac{3}{\pi h_{\max}} \exp \left[-\frac{(\delta - h_{\max})^2}{h_{\max} \cdot 3 / \sqrt{2}} \right]. \quad (8)$$

As has been indicated, the contacting surfaces form a set of closely packed elementary volumes on the surface, differing in the value of the gap δ . It is now assumed that the number of volumes with a gap δ lying in the range $\delta_1 \leq \delta < \delta_2$ is proportional to the probability that the random quantity δ falls in the interval $[\delta_1; \delta_2]$. The thermal conductivity α , which is functionally related to δ , is evidently also a random quantity, with the same probability density $g(\delta)$. In this case, the mean thermal conductivity is

$$\alpha_m = \int_0^{h_1+h_2} \alpha(\delta) g(\delta) d\delta. \quad (9)$$

Or, passing to a finite interval, it is

$$\alpha_m = \sum_{i=1}^n P_i \alpha_i, \quad (10)$$

where n is the number of divisions; P_i , probability that a cell with a definite δ_i will appear; α_i , thermal conductivity of the i -th cell.

Taking the confidence range $\pm 3\sigma$, and determining the thermal conductivity at the midpoint of each of six intervals of width σ , specifically $\alpha_1, \alpha_2, \dots, \alpha_6$, a stepwise-linearized approximate formula for α_m may be obtained in the form

$$\alpha_m = 0.02(\alpha_1 + \alpha_6) + 0.34(\alpha_3 + \alpha_4) + 0.14(\alpha_2 + \alpha_5). \quad (11)$$

Mathematical Model of Contact and Its Realization

In Fig. 2 the form of the current-line distribution and the form of the isotherm in an elementary volume are shown for the case when $\lambda_p > \lambda_m$, the side surface at $r = a$ is adiabatic, and the surfaces at $z = \pm L$ are isothermal. Denoting the temperatures of the surfaces $z = \pm L$ by t' and t'' , and the heat flux by Q , the definition of the thermal resistance R of the volume is then

$$R = \frac{t' - t''}{Q} \quad (12)$$

The calculation of the thermal resistance reduces to the analysis of the temperature field of the system of bodies. The temperatures in the regions occupied by the first and second segments are denoted by t_1 and t_3 , and that in the intercontact region by t_2 . The temperature field of these regions is described by differential equations of the form

$$\frac{\partial^2 t_j}{\partial x^2} + \frac{1}{x} \frac{\partial t_j}{\partial x} + \frac{\partial^2 t_j}{\partial y^2} = 0, \quad j = 1, 2, 3,$$

with the boundary conditions

$$\begin{aligned} \left. \frac{\partial t_j}{\partial x} \right|_{x=0} &= 0; \quad \lambda_i \left. \frac{\partial t_i}{\partial n} \right|_{y=f(x)} = \lambda_2 \left. \frac{\partial t_2}{\partial n} \right|_{y=f(x)}; \quad t_i|_{y=f(x)} = t_2|_{y=f(x)}; \\ t_i|_{y=\pm L} &= \text{const}; \quad \left. \frac{\partial t}{\partial x} \right|_{x=a} = 0, \quad i = 1, 3. \end{aligned}$$

According to the model adopted, the equations of the surfaces take the form

$$f_1(x) = L + r_1^2 - h_1 - \sqrt{r_1^2 - x^2}; \quad f_2(x) = h_2 - L - r_2 + \sqrt{r_2^2 - x^2}.$$

Determining the temperatures t_j , the flux through an elementary volume Q may be found

$$Q = -\lambda \int_S \left. \frac{\partial t_1}{\partial y} \right|_{z=L} dS = -2\pi\lambda_1 \int_0^a \left. \frac{\partial t_1}{\partial z} \right|_{z=L} x dx.$$

This problem is solved using the approximate method of calculating the generalized conductivity outlined in [9]. The basic feature of the method is that the curvilinear current lines are replaced by rectilinear lines, which simplifies the mathematical description of the investigated processes. To linearize the flux in an elementary volume, it is divided by a system of auxiliary adiabatic surfaces parallel to the flux and isothermal surfaces perpendicular to the flux.

The solution obtained by this method has been compared with the results of the standard solution. The standard solution is taken to be a numerical solution of the problem of contact at a point between two hemispheres [10]. Comparison with the results of numerical solution shows that the true value of the resistance is satisfactorily described by the dependence

$$R = (R_a + R_i)/2,$$

where R_a and R_i are the resistances for the adiabatic and isothermal divisions.

Since the problem may be regarded as symmetric with respect to the plane dividing the elementary cell in half, the calculation may be carried out for half the cell. The thermal resistance of an annular layer (current tube) of thickness dx with an adiabatic side surface parallel to the general direction of the heat flux is

$$dR_a = dR_1 + dR_2, \quad (13)$$

where dR_1 and dR_2 are the resistances of the sections of the layer with thermal conductivity λ_1 and λ_2 . Writing dR_i ($i = 1, 2$) in the form

$$dR_1 = \frac{\sqrt{r^2 - x^2} - (r - h)}{2\pi\lambda_1 x dx}; \quad dR_2 = \frac{L - [\sqrt{r^2 - x^2} - (r - h)]}{2\pi\lambda_2 x dx} \quad (14)$$

and substituting Eq. (14) into Eq. (13), an expression for dR_a is obtained

$$dR_a = \frac{1}{2\pi x dx} \left\{ \frac{\lambda_2 [\sqrt{r^2 - x^2} - (r - h)] + \lambda_1 (L - [\sqrt{r^2 - x^2} - (r - h)])}{\lambda_1 \lambda_2} \right\}. \quad (15)$$

Integrating Eq. (15) with respect to x from 0 to a , an expression is obtained for the thermal resistance at the adiabatic division

$$\begin{aligned} R_a &= A_a \frac{(1 - v)}{2\pi\lambda_2 r}; \quad c = \sqrt{1 - \left(\frac{a}{r}\right)^2}; \quad g = \frac{L/r}{1 - v} - \frac{h}{r} + 1; \\ v &= \frac{\lambda_2}{\lambda_1}; \quad A_a = \left[c + g \ln \frac{g - c}{g - 1} - 1 \right]^{-1}. \end{aligned} \quad (16)$$

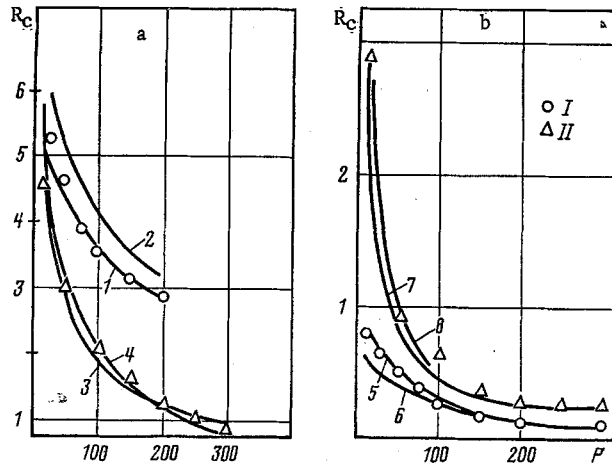


Fig. 3. Dependence of contact thermal resistance $R_c \cdot 10^{-4} \text{ m}^2 \cdot \text{K/W}$ on the air pressure $P \cdot 10^5 \text{ N/m}^2$: a) results for the material 1Kh18N9T, purity of treatment $\nabla 5 - \nabla 5$, from experiment (I), from Eq. (21) (1), and from the method of [1] (2), and results for the material St.45 vapor, purity of treatment $\nabla 4 - \nabla 7b$, from experiment (II), from Eq. (21) (3), and from the method of [1] (4); b) results for the material D16 vapor, purity of treatment $\nabla 8 - 8c$ from experiment (I), from Eq. (21) (5) and from the method of [1] (6) and another set of results from experiment (II), from Eq. (21) (7), and from the method of [1] (8).

The resistance of a plane layer of thickness dy with an isothermal base perpendicular to the heat flow takes the form

$$dR_i = \frac{dR_3 dR_4}{dR_3 + dR_4}, \quad (17)$$

where dR_3 and dR_4 are the resistances of sections of the layer with thermal conductivity λ_1 and λ_2 .

It is obvious that

$$dR_3 = \frac{dy}{\lambda_1 \pi [r^2 - (y + r - h)^2]}; \quad dR_4 = \frac{dy}{\lambda_2 \pi [a^2 - r^2 + (y + r - h)^2]}. \quad (18)$$

Substituting dR_3 and dR_4 from Eq. (18) into Eq. (17), an expression for dR_i is obtained, and integrating this with respect to y allows the thermal resistance R_i to be determined

$$dR_i = \frac{dy}{\pi [\lambda_1 r^2 + \lambda_2 (a^2 - r^2) - (\lambda_1 - \lambda_2) (y + r - h)^2]}; \quad (19)$$

$$R_i = \frac{\ln k + m}{2\pi b (1 - \nu) \lambda_1}; \quad m = 2 \frac{Lb}{a^2} \left(1 - \frac{h}{L}\right) \left(\frac{1 - \nu}{\nu}\right); \quad (20)$$

$$b = \sqrt{r^2 - \frac{\nu}{1 - \nu} a^2}; \quad k = \frac{\left(\frac{b}{r} + 1\right) \left(\frac{b}{r} - 1 + \frac{h}{r}\right)}{\left(\frac{b}{r} - 1\right) \left(\frac{b}{r} + 1 - \frac{h}{r}\right)}.$$

Thus, Eqs. (16) and (20) allow the admittedly increased R_a and reduced R_i values of the resistance of half the cell to be calculated. Finally, the resistance of the cell is taken to be the arithmetic mean, as indicated earlier. Then for the whole cell

$$R = (R_a + R_i)/2. \quad (21)$$

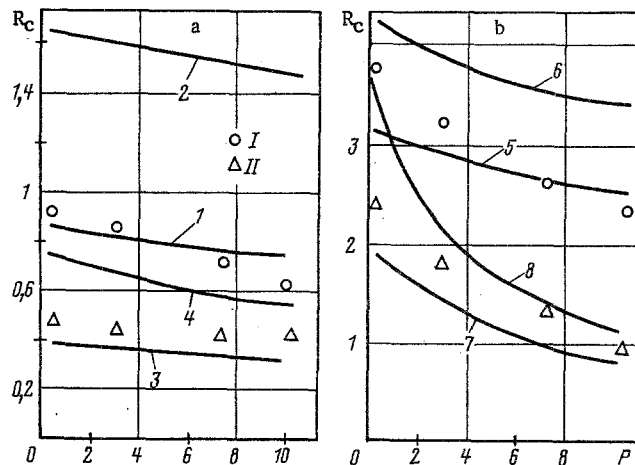


Fig. 4. Dependence of contact thermal resistance $R_c \cdot 10^{-4} \text{ m}^2 \cdot \text{K}/\text{W}$ on pressure $p \cdot 10^5 \text{ N}/\text{m}^2$: a) results in the contact zone of PFMS-4 resin for the mat material St.45 vapor, microprojection height $h = 20 \mu$, from experiment (I), from Eq. (21) (1), and from the method of [1] (2), and results for the material D16T vapor, height = $10 \mu\text{m}$, from experiment (II), from Eq. (21) (3), and from the method of [1] (4); b) results in the contact zone of air, for the material St.45 vapor, $h = 12 \mu\text{m}$, from experiment (I), from Eq. (21) (5), and from the method of [1] (6); and for the material D16T vapor, $h = 10 \mu\text{m}$, from experiment (II), from Eq. (21) (7), and from the method of [1] (8).

Analysis of Solutions

To verify the model proposed and the corresponding theoretical relations, calculations by this method were compared with the experimental results of [1] and also with calculations by the method adopted in [1]. In comparing the experimental and theoretical results of [1], the relative error is in the range $\delta_p = \pm 33\%$ with a confidence level of 0.95. The mean square deviation here is 14%. In comparing the experiment [1] with calculations by the method here proposed, the relative error is in the range $\delta_p = \pm 15\%$, with a confidence level of 0.95. The mean square deviation is 7%. In Figs. 3 and 4, experimental and theoretical pressure dependences of the CTR are given.

In comparing the present experiments with calculations by the method of [1], the relative error lies in the range $\delta_p = \pm 32\%$ with a confidence level of 0.95. The mean-square deviation is 15%. In comparing the present experiments with calculations by the method here proposed, the relative error lies in the range $\delta_p = \pm 15\%$. The mean square deviation is 7%. In the whole experiment the relative error in the calculations according to [1] did not exceed 35%, and in calculations by the method proposed here - 19%.

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THEORETICAL AND EXPERIMENTAL INVESTIGATION
OF HIGH-LEVEL RADIATION SOURCES USED TO
MODEL A HEAT INPUT

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This paper examines high-intensity xenon-filled radiation sources for heat load simulation. A mathematical model of the discharge is proposed, and results of a theoretical and an experimental investigation are presented.

One of the main problems which one has to resolve in various branches of technology is that of testing objects under conditions as close as possible to the actual conditions. Many literature references (a detailed bibliography is given in [1]) discuss problems associated with creating facilities to simulate conditions of operation of items of contemporary technology. In some cases a key element of such simulation systems is a high-intensity radiative source (HIRS). A promising HIRS is the pulsed xenon source which has high efficiency in converting electrical into radiative energy, is economical, has a large radiative flux density and high operating stability. To construct a high-efficiency HIRS and to predict its properties reliably requires theoretical and experimental investigations.

The present paper gives some results of mathematical modeling of the working process in a HIRS, determines the limiting characteristics, and describes an experimental investigation. It should be noted that in natural-model thermal endurance tests an HIRS may operate in a heater system comprising several sources and reflectors. The investigation and the development of an HIRS incorporated into a facility requires the development of a complete system for computer design of such facilities, based on methods of designing and testing of an individual isolated HIRS.

Mathematical Model of the Discharge. We assume that the plasma is in a state of local thermodynamic equilibrium (LTE). The system of energy balance and radiative transfer equations describing the physical processes in the discharge takes the form

$$\frac{1}{R^2 z} \frac{d}{dz} \left[z \lambda_r(T) \frac{dT}{dz} \right] + \sigma(T) E_1^2 - \operatorname{div} F_r = 0, \quad (1)$$

$$\frac{1}{3R^2 z} \frac{d}{dz} \left[z \frac{1}{K_{v\Sigma}} \frac{du_v}{dz} \right] + K'_{v\Sigma} (u_{ve} - u_v) = 0, \quad (2)$$

$$\operatorname{div} F_r = c \int_0^\infty K'_{v\Sigma} (u_{ve} - u_v) dv. \quad (3)$$

The boundary conditions are

$$z = 0, \quad \frac{dT}{dz} = 0, \quad \frac{du_v}{dz} = 0, \quad (4)$$

$$z = 1, \quad T = T_w, \quad u_v = -\frac{A}{RK'_{v\Sigma}} \frac{du_v}{dz}. \quad (5)$$

Here we neglect convection and postulate that the voltage E_1 is constant along the arc. Equations (2) and (3) describe radiative transfer in the diffusion approximation. According to the data of [2], the constant $A = 0.847$.

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